Relation between the phonation threshold pressure and the prephonatory glottal width in a rectangular glottis

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Recent experimental measurements have shown a nonlinear relation between the phonation threshold pressure and the prephonatory glottal width, with a minimum for the threshold pressure which would indicate the existence of an optimal glottal width for ease of phonation [I. R. Titze *et al.*, J. Acoust. Soc. Am. **97**, 3080–3084 (1995)]. This relation is studied analytically using a simplified vocal fold model which includes an explicit term for the air pressure losses due to glottal viscous resistance. It is shown that the observed nonlinearity may be a consequence of the viscous pressure losses, which cause an increase of the threshold pressure at small values of the prephonatory glottal width. © *1996 Acoustical Society of America.*

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INTRODUCTION

The phonation threshold pressure, defined as the minimum lung pressure required to initiate the vocal fold oscillation, has been studied in recent works from both theoretical and experimental approaches (Titze, 1988, 1989, 1992; Titze *et al.*, 1995; Lucero, 1995). An analytical expression in terms of the glottal geometry and biomechanical parameters was derived by Titze (1988), assuming a surface wave propagating through the cover of the vocal folds and small-amplitude conditions. The expression predicted a linear increase of the threshold pressure with the prephonatory glottal width. Its validity was later tested in experimental measurements by Titze *et al.* (1995) on a physical model of the vocal fold mucosa, where the air pressure to initiate the mucosa oscillation was measured for various glottal conditions. The results confirmed in general the analytical predictions; however, it was found that the threshold pressure did not decrease to zero as the glottal width was reduced. Instead, a nonlinear relation with a minimum at a positive glottal width was observed.

Since the threshold pressure may be considered as a measure of ease of phonation, a minimum value would mean the existence of an optimal glottal width (i.e., a glottal width at which the effort required to initiate phonation is minimum), as noted by Titze *et al.* (1995). A possible explanation for the nonlinear relation between threshold pressure and glottal width was offered in their paper, attributing it to a glottal closure in part of the oscillatory cycle. However, since the vocal folds are separated at the start of their oscillation, the threshold condition could not be influenced by glottal closure. Mathematically, the oscillation threshold corresponds to a Hopf bifurcation, at which a stable equilibrium position becomes unstable and a limit cycle is produced (Lucero, 1993). Since the stability properties of an equilibrium position are determined only by local conditions (i.e., in small neighborhood of the equilibrium position), the oscillation threshold must be described only by the open glottis dynamics. Thus an alternative explanation must be sought.

In this Letter we will investigate the influence of the glottal viscous resistance, which was not considered in the previous analyses. Note that the difference between the analytical prediction and the experimental result appears at low values of the glottal width, precisely where pressure losses by viscous resistance become significant; hence, a relation between them is possible. We will thus extend Titze's analysis including a viscous resistance term in his equations, and will then re-examine the threshold condition.

I. VOCAL FOLD MODEL

The vocal fold model is shown in Fig. 1 (Titze, 1988). The vocal fold body is stationary, and the cover propagates a surface wave in the direction of the airflow. Assuming a small time delay of the surface wave in moving from the lower to the upper edges of the vocal fold, and lumping the biomechanical properties of the vocal fold at the midpoint of the glottis, we obtain the equation of motion

$$M\ddot{\xi}+B\dot{\xi}+K\xi=P_g, \tag{1}$$

where M, B, and K are the lumped effective mass, damping, and stiffness per unit area of the cover, respectively, ξ is the lateral displacement of the cover at the midpoint, and P_g is the glottal pressure acting on the vocal fold (Titze, 1988).

In his analysis, Titze considered for the transglottal pressure the expression

$$P_s-P_i=k_t\rho u^2/2a_2^2, \tag{2}$$

where P_s is the subglottal pressure, P_i is the pressure at the vocal tract input, kt is an empirical coefficient related to kinetic pressure losses for the formation of a vena contracta at the glottal entry and turbulent pressure recovery at the glottal exit, ρ is the air density, u is the air volume flow velocity, and a_2 is the glottal area at the upper edge of the vocal fold. The glottal viscous resistance was considered to be part of the coefficient k_t.

Here we will add an explicit term for the viscous resistance. From experimental measurements on a cast model of the larynx at steady flow condition, van den Berg *et al.* (1957) observed a good agreement between the measured viscous pressure loss and the Poiseuille formula

$$\Delta P_v=12\mu L^2Tu/a^3, \tag{3}$$

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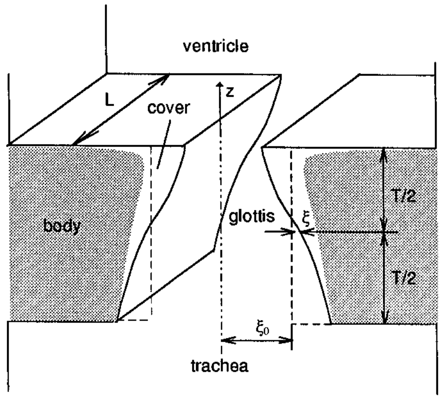

FIG. 1. Vocal fold model. Broken line: prephonatory position. T: vocal fold thickness. L: vocal fold length. ξ_0: prephonatory displacement. ξ: displacement at the midpoint of the vocal fold (Titze, 1988).

where μ is the air viscosity, L is the vocal fold length, T is it thickness, and a is the glottal area, which was constant along the direction of the airflow.

In our model the glottal area varies in time and along the glottis; it is not clear how Eq. (3) should be modified for these conditions. We assume first a rectangular prephonatory glottis, i.e., a constant glottal area along the airflow direction. Note that the experimental results obtained by Titze *et al.* (1995) were also obtained with a rectangular prephonatory glottis. We also assume small amplitude oscillations, since we are interested only in the threshold conditions. Under these assumptions, the glottis is approximately rectangular during the vocal fold oscillation; we will use then Eq. (3) as an approximation taking the glottal area a as the area at the midpoint of the glottis. Equation (2) is hence modified to

$$P_s-P_i=\frac{k_t\rho u^2}{2a_2^2}+\frac{12\mu L^2Tu}{a^3}. \tag{4}$$

The glottal pressure P_g acting on the vocal folds is next calculated as the mean glottal pressure along the glottis (Titze, 1988)

$$P_g=\frac{1}{T}\int_{-T/2}^{T/2}P(z)dz, \tag{5}$$

where $P(z)$ is the intraglottal pressure, and the origin of the z axis has been taken at the midpoint of the glottis. The intraglottal pressure may be expressed by

$$P(z)=P_2+\frac{\rho u^2}{2a_2^2}\left(1-\frac{a_2^2}{a^2(z)}\right)+\frac{12\mu L^2u(T/2-z)}{a^3}, \tag{6}$$

where P_2 is the exit pressure (Titze, 1988) and $a(z)$ is the intraglottal area. The last term was added to the original equation in Titze's paper assuming that the viscous losses vary linearly along the glottis [note in Eq. (3) that the viscous losses are proportional to the glottal thickness T].

The assumption of a small time delay for the surface wave plus a linear variation of the prephonatory glottal area along the glottis implies a linear variation along the glottis also for the time-varying glottal area (Lucero, 1995), i.e.,

$$a(z)=a_1+\frac{a_2-a_1}{T}\left(z+\frac{T}{2}\right), \tag{7}$$

where a_1 is the glottal area at the lower edge of the vocal fold. Introducing Eqs. (6) and (7) into Eq. (5) and integrating, we obtain

$$P_g=P_2+\frac{\rho u^2}{2a_2^2}\left(1-\frac{a_2}{a_1}\right)+\frac{6\mu L^2Tu}{a^3}. \tag{8}$$

The above equations may be simplified further assuming as in Titze (1988) that the subglottal pressure P_s is constant and equal to the lung pressure P_L, the vocal tract input pressure is equal to the atmospheric pressure ($P_i=0$), and the supraglottal area is large compared with a_2, which implies $P_2=P_i$. The first two assumptions neglect the loading effect of the subglottal system and the vocal tract, and might seem a crude approximation. They are introduced to eliminate parameters and reduce the model to the basic vocal fold dynamics, following previous analytical studies of the vocal fold oscillation (e.g., Titze, 1988; Lucero, 1993, 1995; Steinecke and Herzel, 1995). Thus Eqs. (4) and (8) become

$$P_L=\frac{k_t\rho u^2}{2a_2^2}+\frac{12\mu L^2Tu}{a^3}, \tag{9}$$

$$P_g=\frac{\rho u^2}{2a_2^2}\left(1-\frac{a_2}{a_1}\right)+\frac{6\mu L^2Tu}{a^3}. \tag{10}$$

The dynamics of the vocal folds is therefore expressed by Eqs. (1), (9), and (10), together with the equations for the glottal areas

$$a_1=2L(\xi_0+\xi+\tau\dot{\xi}), \tag{11}$$

$$a_2=2L(\xi_0+\xi-\tau\dot{\xi}), \tag{12}$$

$$a=2L(\xi_0+\xi), \tag{13}$$

where ξ_0 is the prephonatory displacement (equal for the upper and lower edges), and

$$\tau=T/2c \tag{14}$$

is the time delay of the surface wave from the lower edge of the vocal folds to the midpoint of the glottis or from the midpoint to the upper edge, and c is the surface wave velocity.

II. THRESHOLD CONDITIONS

A. Large glottal area

The oscillation threshold condition represents the parameters at which the equilibrium position of the vocal folds becomes unstable.

When the glottal area is large, the pressure loss for glottal viscous resistance is small and the viscous terms in Eqs. (9) may be neglected. Solving for the volume flow velocity u we obtain

$$u=a_2\sqrt{2P_L/k_t\rho}. \tag{15}$$

Introducing this expression into Eq. (10), it becomes

$$P_g = \frac{P_L}{k_t}\left(1-\frac{a_2}{a_1}\right) + \frac{6\mu L^2 T a_2}{a^3}\sqrt{\frac{2P_L}{k_t\rho}}$$

$$= \left(\frac{P_L}{k_t}\right)\frac{2\tau\dot{\xi}}{\xi_0+\xi+\tau\dot{\xi}} + \frac{3\mu T(\xi_0+\xi-\tau\dot{\xi})}{2(\xi_0+\xi)^3}\sqrt{\frac{2P_L}{k_t\rho}} \quad (16)$$

[note that the viscous term in Eq. (10) may be not neglected, because the first term can be smaller depending on the values of a_1 and a_2, e.g., it is zero for $a_1 = a_2$].

The equilibrium position of the vocal fold is determined by setting to zero the time derivatives in Eqs. (1) and (16), with the result

$$K\bar{\xi} = \frac{3\mu T}{2(\xi_0+\bar{\xi})^2}\sqrt{\frac{2P_L}{k_t\rho}}, \quad (17)$$

where $\bar{\xi}$ denotes the equilibrium displacement.

The threshold condition is obtained next linearizing Eq. (16) around $\xi = \bar{\xi}$, $\dot{\xi} = 0$, which yields the linear equation of motion

$$M\ddot{x} + \left(B - \frac{\partial P_g}{\partial \dot{\xi}}(\bar{\xi},0)\right)\dot{x} + \left(K - \frac{\partial P_g}{\partial \xi}(\bar{\xi},0)\right)x = 0, \quad (18)$$

where $x = \xi - \bar{\xi}$ is a small deviation from the equilibrium, and setting the coefficient of $\dot{\xi}$ (the effective damping) equal to zero (Titze, 1988; Lucero, 1995); i.e., at threshold

$$B = \frac{\partial P_g}{\partial \dot{\xi}}(\bar{\xi},0). \quad (19)$$

From Eq. (16) we have

$$\frac{\partial P_g}{\partial \dot{\xi}}(\bar{\xi},0) = \frac{2\tau P_L}{k_t(\xi_0+\bar{\xi})} - \frac{3\mu T\tau}{2(\xi_0+\bar{\xi})^3}\sqrt{\frac{2P_L}{k_t\rho}} \quad (20)$$

and introducing this equation into Eq. (19) and solving for P_L, we obtain

$$P_{\mathrm{th}} = \left(\frac{3\mu T}{8(\xi_0+\bar{\xi})^2}\sqrt{\frac{2}{k_t\rho}} + \sqrt{\frac{9\mu^2T^2}{32k_t\rho(\xi_0+\bar{\xi})^4} + \frac{k_tB(\xi_0+\bar{\xi})}{2\tau}}\right)^2, \quad (21)$$

where P_{th} denotes the threshold value of the lung pressure.

When the prephonatory displacement ξ_0 is large, then the viscous terms in Eq. (21) may be neglected. Also, from Eq. (17) we see that the equilibrium displacement $\bar{\xi}$ becomes smaller as ξ_0 increases, and may also be neglected for ξ_0 large enough. In this case, Eq. (21) may be simplified to

$$P_{\mathrm{th}} = k_t B\xi_0/2\tau, \quad (22)$$

which is the equation for the threshold pressure obtained by Titze (1988). We can see that the threshold pressure increases linearly with ξ_0.

As the ξ_0 reduces, the viscous terms in Eq. (21) become dominant. Since ξ_0 appears in their denominators, then the threshold pressure will eventually reach a minimum and will increase for lower values of ξ_0. It seems difficult to obtain an analytical expression for the value of ξ_0, where P_{th} takes its minimum; note that the equilibrium displacement $\bar{\xi}$ in Eq. (21) is also a function of ξ_0 and P_L, as expressed by Eq. (17). The minimum in relation to $(\xi_0+\bar{\xi})$ its easier to obtain, setting to zero the derivative of P_{th} in relation to $(\xi_0+\bar{\xi})$ and solving, with the result

$$\xi_0 + \bar{\xi} = \sqrt[5]{27\tau\mu^2T^2/2\rho B}. \quad (23)$$

The value of ξ_0 in Eq. (23) represents then the optimal glottal width for ease of phonation. It decreases when the vocal fold damping B is increased, in agreement with the experimental results presented by Titze *et al.* (1995).

B. Small glottal area

When ξ_0 is even closer to zero, the viscous term in Eq. (9) becomes larger than the first term (note that the former is proportional to a^{-3}, whereas the latter is proportional to a_2^{-2}), which may be then neglected. Solving for the volume flow velocity u we now obtain

$$u = P_L a^3/12\mu L^2 T \quad (24)$$

and introducing this result into Eq. (10)

$$P_g = \frac{\rho P_L^2 a^6}{288\mu^2 L^4 T^2 a_2^2}\left(1-\frac{a_2}{a_1}\right) + \frac{P_L}{2}$$

$$= \frac{\rho\tau P_L^2\dot{\xi}(\xi_0+\xi)^6}{9\mu^2T^2(\xi_0+\xi-\tau\dot{\xi})^2(\xi_0+\xi+\tau\dot{\xi})} + \frac{P_L}{2}. \quad (25)$$

As before, we determine first the equilibrium displacement setting the time derivatives to zero, with the result

$$\bar{\xi} = P_L/2K. \quad (26)$$

From Eq. (26), we obtain next

$$\frac{\partial P_g}{\partial \dot{\xi}}(\bar{\xi},0) = \frac{\rho\tau P_L^2(\xi_0+\bar{\xi})^3}{9\mu^2T^2} \quad (27)$$

and the threshold condition becomes

$$P_{\mathrm{th}} = 3\mu T\sqrt{B/\tau\rho(\xi_0+\bar{\xi})^3}. \quad (28)$$

This equation shows that the threshold pressure increases as ξ_0 becomes close to zero.

III. NUMERICAL EXAMPLES

The plots in Figs. 2 and 3 (curve 1) show the values of the equilibrium displacement $\bar{\xi}$ at threshold and the threshold pressure P_L versus the prephonatory displacement ξ_0. They were obtained approximating first the differential equation (19) by

$$B \cong \frac{P_g(\bar{\xi},\Delta\dot{\xi}) - P_g(\bar{\xi},0)}{\Delta\dot{\xi}}, \quad (29)$$

where $\Delta\dot{\xi}$ denotes a small value. The value of P_L which satisfies this equation was next determined by a standard numerical method. At the same time, the value of the equilibrium displacement $\bar{\xi}$ needed in the above equation was obtained solving numerically Eqs. (1), (9), and (10), with the

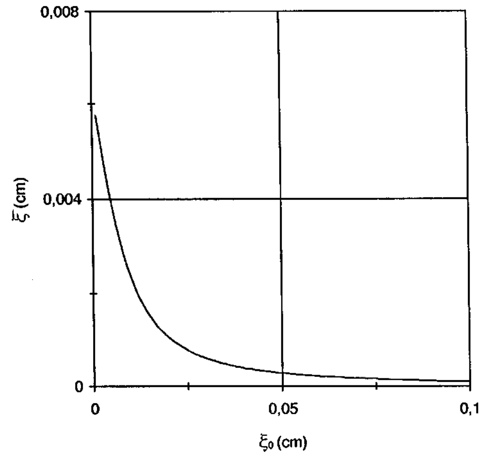

FIG. 2. Equilibrium displacement at the phonation threshold pressure versus prephonatory displacement.

time derivatives equal to zero. The parameters of the model were given the values $T=0.3$ cm, $L=1.4$ cm, $c=100$ cm/s, $k_t=1.1$, $B=0.1$ N s/m (Titze, 1988), $\rho=1.14$ kg/m^3, and $\mu=1.86\times10^{-5}$ kg/(s m) (Ishizaka and Flanagan, 1972). A value of $\Delta\dot{\xi}=10^{-3}$ cm/s was adopted for Eq. (29).

In Fig. 2 we can see that for large values of ξ_0 the equilibrium displacement is close to zero, and increases monotonically as ξ_0 decreases. This is a consequence of the viscous term in the glottal pressure, given by Eq. (10). This term adds a positive component to the glottal pressure which tends to push the vocal folds apart, and is in inverse relation to the glottal area.

In curve 1 of Fig. 3 we can see a nonlinear relation between the threshold pressure and the prephonatory displacement, with a minimum value for the threshold pressure. The shape of the curve is identical to those obtained experimentally by Titze *et al.* (1995). At large values of the prephonatory displacement, the threshold pressure increases almost linearly. As the prephonatory displacement decreases, the slope of the curve reduces and a minimum occurs at $\xi_0=0.018$ cm. It is interesting to note that this value is the prephonatory displacement adopted for the standard configuration of the two-mass model of the vocal folds (Ishizaka and Flanagan, 1972). Below this value the threshold pressure increases as the prephonatory displacement becomes closer to zero.

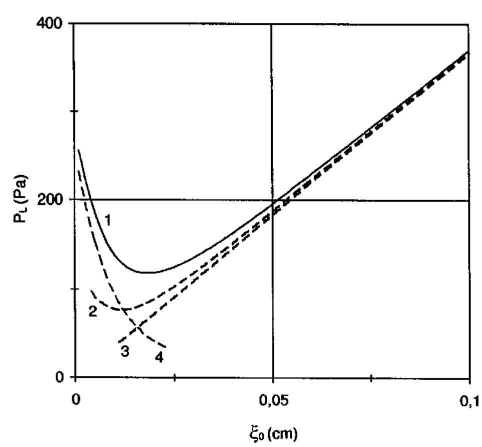

FIG. 3. Phonation threshold pressure versus prephonatory displacement. Curve 1: numerical solution of Eq. (19). Curve (2): approximation in Eqs. (17) and (21). Curve (3): linear approximation in Eq. (22). Curve (4): approximation in Eqs. (26) and (28).

Values obtained using the various expressions derived above were also plotted in Fig. 3, as an internal validation of the analysis. Curve (2) was obtained solving Eqs. (17) and (21), which are the approximations for large values of ξ_0. Curve (3) is the linear approximation in Eq. (22), derived previously by Titze (1988). Curve (4) is the approximation for ξ_0 close to zero, from Eqs. (26) and (28).

We can also see in Fig. 2 that at the minimum threshold pressure, the equilibrium displacement is $\xi=0.0012$ cm, much smaller than the prephonatory displacement. The equilibrium displacement in Eq. (23) may be then neglected to facilitate the calculations, which results in the simpler expression for the optimal prephonatory displacement.

$$\xi_0 \cong \sqrt[5]{27\tau\mu^2T^2/2\rho B}. \tag{30}$$

Using the above values, we obtain from this equation $\xi_0=0.014$ cm, close to the value obtained by numerical technique.

IV. CONCLUSION

This analysis tends to show that viscous pressure losses at the glottis are important for describing the nonlinear relation between the phonation threshold pressure and prephonatory glottal width found by Titze *et al.* (1995). A more extensive study of the threshold condition would be interesting as a next step, considering the general case of convergent and divergent glottal shapes and more detailed flow models including flow inertia and flow separation effects, as suggested by one of the reviewers of this Letter.

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