Chest- and falsetto-like oscillations in a two-mass model of the vocal folds

Jorge C. Lucero[a)]
Department of Computer Science, University of Brasilia, Brasilia DF 70910-900, Brazil

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The dynamics of the two-mass model of the vocal fold oscillation is analyzed. It is shown that the oscillation may occur around two equilibrium positions, and that each case presents similar features as the chest and falsetto registers, suggesting a relation between them. The switch between equilibrium positions is caused by a transcritical bifurcation phenomenon. © *1996 Acoustical Society of America.*

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INTRODUCTION

The nonlinear dynamics of the vocal fold oscillation was analyzed in a previous work using a simplified version of the two-mass model (Lucero, 1993). Two equilibrium positions besides the rest position were found for the vocal folds, and a bifurcation diagram was derived for the case of a rectangular glottis at the rest position. It was shown that the oscillation region is divided into two subregions by a transcritical bifurcation, and that in each subregion the oscillation occurs around different equilibrium positions. These results were a novel finding for the oscillation dynamics, and have been confirmed in later works (Steinecke and Herzel, 1995). However, their meaning in terms of phonation has not been clarified yet.

In this paper we will present further analytical results which suggest that each of the subregions of oscillation could be associated to a different vocal register, chest and falsetto.

Let us briefly recall that the vocal fold may be divided into two loosely coupled layers with different mechanical properties: body and cover (Titze, 1988, 1994; Story and Titze, 1995). The body consists of muscle and deep layers of the vocal ligament and forms the bulk of the fold, whereas the cover consist of the more superficial tissues around the body. During the oscillation, the body undergoes lateral motion, and the cover propagates a surface wave in the direction of the airflow. The combined motion of body and cover may be also regarded as a lateral oscillation of the vocal folds with a vertical phase difference, or time delay from the bottom to the top of the fold.

In normal phonation at the chest register, the stiffness of the body of the vocal folds is much larger than the stiffness of the cover. The oscillation occurs with large amplitude and vertical phase difference, collision between the opposite vocal folds, and a near rectangular or even divergent glottal shape. In the falsetto register, on the other hand, the stiffness of the cover becomes larger and in the order of the body stiffness. In this case the oscillation is nearly sinusoidal without fold collision and a small phase difference, and with a convergent glottal shape (Titze, 1994).

We will show in the following analysis that the oscillation of the two-mass model around each equilibrium position presents the same features as the vocal registers, which suggests a relation between them.

I. THE TWO-MASS MODEL

The two-mass of model of the vocal folds is schematically shown in Fig. 1 (Ishizaka and Flanagan, 1972). Each vocal fold is divided into two single mechanical oscillators coupled through a spring. In the standard configuration, the lower mass is taken thicker and more massive than the upper one, and with a higher stiffness coefficient, as an attempt to represent the effect of the body. Note that the standard values for the lower and upper masses and stiffnesses $m_1=0.125$ g, $m_2=0.025$ dyn/cm, $k_1=80\,000$ dyn/cm, and $k_2=8000$ dyn/cm, are close to and keep the same ratio as those derived by Story and Titze (1995) from experimental results for the body and cover, $m_{\text{body}}=0.05$ g, $m_{\text{cover}}=0.01$ g, $k_{\text{body}}\geqslant 50\,000$ dyn/cm, and $k_{\text{cover}}\geqslant 5000$ dyn/cm. We will then assume that the mechanical properties of the body and cover are roughly represented by the parameters of the lower and upper oscillators, respectively.

As in the previous work, we neglect nonlinearities in the biomechanical properties of the tissues and the influence of the vocal tract and subglottal system. A further simplification of the glottal aerodynamical equations is introduced here, assuming Bernoulli flow below the glottal constriction, and a free downstream jet above it (Steinecke and Herzel, 1995; Story and Titze, 1995). With these assumptions, the equations of motion are

$$\begin{aligned} &m_1\ddot{x}_1+r_1\dot{x}_1+s_1+k_c(x_1-x_2)=F_1,\\ &m_2\ddot{x}_2+r_2\dot{x}_2+s_2+k_c(x_2-x_1)=0, \end{aligned} \tag{1}$$

where x_i $(i=1,2)$ is the displacement of mass m_i from its rest position, r_i is the damping coefficient, k_c is the coupling stiffness, s_i is the elastic restoring force,

$$s_i=\begin{cases} k_ix_i, & \text{for } x_i>-x_{i0}\\ k_ix_i+h_i(x_i+x_{i0}), & \text{for } x_i\leqslant -x_{i0} \end{cases} \quad (i=1,2), \tag{2}$$

where k_i is the stiffness coefficient, h_i is the extra stiffness coefficient introduced by the collision between the opposite folds, $x_i=-x_{i0}$ is the mass displacement where collision oc-

[a)]Electronic mail: lucero@cic.unb.br

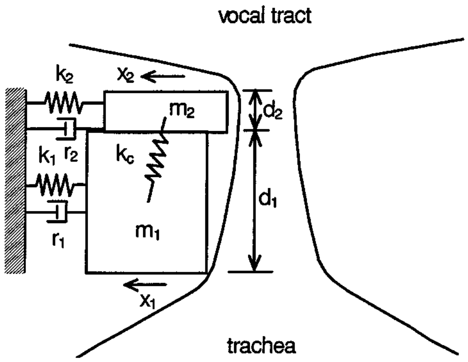

FIG. 1. Two-mass model.

curs, and F_1 is the aerodynamic force on mass m_1

$$F_1=\begin{cases} d_1 l_g P_s\left(1-\dfrac{(x^2+x_{20})^2}{(x_1+x_{10})^2}\right), \quad \text{for } x_1\geqslant x_2, x_1>-x_{10}, \\ x_2>-x_{20}, \\ d_1 l_g P_s, \quad \text{for } x_1>-x_{10}, x_2\leqslant -x_{20}, \\ 0, \quad \text{otherwise}, \end{cases} \tag{3}$$

where d_1 is the width of mass m_1, l_g is its length, and P_s is the subglottal pressure. (A constant pressure equal to P_s is assumed to act on m_1 when collision occurs at m_2.)

For an oscillatory solution to build up from the rest position, the glottis at rest must be convergent, i.e., $x_{10}>x_{20}$. Otherwise, the force F_1 will be zero at the rest position, and the trivial solution $x_1=0$, $x_2=0$, will be a solution of Eq. (1).

II. EQUILIBRIUM POSITIONS

The equilibrium positions appear only in the open glottis condition $x_1>-x_{10}, x_2>-x_{20}$. They may be determined setting the time derivatives to zero in the equations of motion (1), and solving for x_1 and x_2. Two cases must be considered: Assuming a rectangular or convergent glottis at equilibrium ($x_1\geqslant x_2$), we arrive at the simultaneous equations

$$x_2=\alpha x_1, \tag{4}$$

$$(k_1+\alpha k_2)(x_1+x_{10})^3-[x_{10}(k_1+\alpha k_2)+d_1 l_g P_s (1-\alpha^2)](x_1+x_{10})^2-2\alpha d_1 l_g P_s(\alpha x_{10}-x_{20})(x_1+x_{10}) + d_1 l_g P_s(\alpha x_{10}-x_{20})^2=0, \tag{5}$$

where α is the coupling coefficient

$$\alpha=k_c/(k_2+k_c). \tag{6}$$

Second, assuming a divergent glottis at equilibrium ($x_1<x_2$), then the force F_1 is zero and the rest position is the only equilibrium position.

The simpler case of a rectangular glottis at the rest position, $x_{10}=x_{20}=x_0$ was analyzed in our previous work (Lucero, 1993) and also by Steinecke and Herzel (1995). The main results will be reviewed next, for clarity. In this case, Eq. (5) simplifies to

$$x_1[(k_1+\alpha k_2)(x_1+x_{10})^2-d_1 l_g P_s(1-\alpha^2)(x_1+x_{10}) - d_1 l_g P_s x_0(1-\alpha)^2]=0. \tag{7}$$

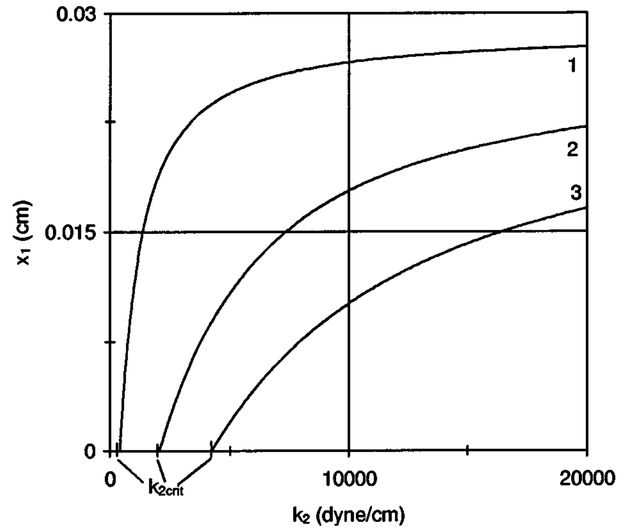

FIG. 2. Coordinate x_1 of the equilibrium positions versus k_2 for a rectangular glottis, and $k_c=1000$ dyn/cm (curve 1), 5000 dyn/cm (curve 2), and 10 000 dyn/cm (curve 3). The equilibrium position at the rest position is located at the x axis ($x_1=0$). The value of $k_{2\text{crit}}$ at which the transcritical bifurcation occurs has been indicated.

This equation has three solutions: $x_1 = 0$, a second one where $x_1\geqslant -x_0$, and a third one where $x_1\leqslant -x_0$. The third solution does not satisfy the open glottis condition and is hence invalid (in Lucero, 1993, a small region where this third solution appeared within the open glottis condition was found. The difference with the present result is caused by the assumption of a free jet above the glottal constriction adopted here. See also Steinecke and Herzel, 1995). Applying next Eq. (4) to the first two solutions, we obtain the coordinates for the equilibrium positions.

From the first solution we obtain $x_1 = 0$ and $x_2 = 0$, which is the rest position of the vocal folds.

Note that Eq. (6) implies that $0\leqslant\alpha<1$, and hence $|x_2|\leqslant|x_1|$ (the equality holds for $\alpha=0$). Since Eq. (7) holds only for a rectangular or convergent glottis at equilibrium, then the second equilibrium position must satisfy the condition $x_1\geqslant x_2\simeq 0$, i.e., at the second equilibrium position the glottis is in general wider than at rest, and convergent.

The two equilibrium positions may become coincident at $x_1=0, x_2=0$. Introducing these values in the second factor of Eq. (7), we obtain

$$P_s=\frac{x_0}{2d_1 l_g}\left(k_1+k_c+\frac{k_1 k_c}{k_2}\right). \tag{8}$$

The stability of each equilibrium position may be determined next by linearizing the equations of motion around each equilibrium position, and studying the roots of the associated characteristic equations. It was shown (Lucero, 1993) that the oscillation region is delimited by two Hopf bifurcations, each one involving a different equilibrium position. Further, the oscillation region is divided into two subregions by a transcritical bifurcation, given by Eq. (8); in each subregion, the oscillation takes place around a different equilibrium position.

We will examine here the effect of the stiffness k_2 on the equilibria discussed above and on their stability. Figure 2

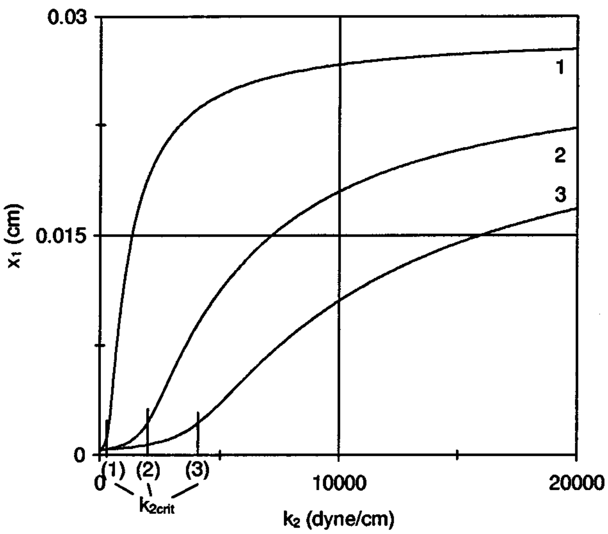

FIG. 3. Coordinate x_1 of the equilibrium position versus k_2 for a slightly convergent glottis, and k_c=1000 dyn/cm (curve 1), 5000 (curve 2), and 10 000 dyn/cm (curve 3). There is no equilibrium position at the rest position, and hence no transcritical bifurcation. The value of k_{2crit} from Fig. 1 has been indicated, as reference.

shows the x_1 coordinate of the equilibrium positions versus k_2, for k_1=80 000 dyn/cm, P_s=8000 dyn/cm^2, l_g=1.4 cm, d_1=0.25 cm, x_0=0.02 cm (Ishizaka and Flanagan, 1972), and k_c as parameter. Below a certain value of k_2 (we will denote this value by k_{2crit}) there is only one equilibrium value of x_1 (x_1=0), and above that value a second solution besides $x_1=0$ appears. The value of k_{2crit} is the transcritical bifurcation given by Eq. (8).

Figure 3 shows results for a slightly convergent glottis, with $x_{10}=0.02$ cm and $x_{20}=0.0199$ cm. All the other parameters in the model were the same as those used in Fig. 2. In this case, the equilibrium positions were calculated directly from Eqs. (4) and (5). Equation (4) has three solutions, however only one of them satisfies the condition $x_1 \geqslant x_2 \geqslant 0$. We have then only one equilibrium position and consequently no transcritical bifurcation; however, the curves have equivalent shapes to those in Fig. 2. The values of k_{2crit} calculated for Fig. 2 have been also indicated for reference. Note that below k_{2crit} the equilibrium position is close to the rest position, and above it becomes farther as k_2 increases.

We may define the glottal convergence angle as

$$\theta=\tan^{-1}[(x_1-x_2)/d]=\tan^{-1}[x_1(1-\alpha)/d], \qquad (9)$$

where d is the total vocal fold thickness. Then, we have that as k_2 increases, x_1 increases as shown in Figs. 2 and 3, and at the same time α decreases [see Eq. (6)]. Both variations contribute to increase the glottal convergence angle.

III. DISCUSSION CONSIDERING VOCAL REGISTERS

According to these results, the oscillation region is divided into two subregions: When the cover stiffness k_2 is lower than the transcritical value k_{2crit}, the oscillation takes place around an equilibrium position in which the glottis is rectangular or slightly convergent. Above the transcritical value k_{2crit}, the equilibrium position switches to a different location where the glottis is wider and more convergent. The switch between the equilibrium positions becomes more abrupt at lower values of the coupling stiffness k_c.

Considering the characteristics of the chest and falsetto registers mentioned in the introductory section, we could associate the subregion $k_2<k_{2crit}$ to the chest register, and $k_2>k_{2crit}$ to the falsetto.

Let us consider how the switch between registers may occur. The stiffness of the vocal folds is controlled mainly by action of the crycothyroid (CT) and the thyroarytenoid (TA) muscles (Titze *et al.*, 1988; Titze, 1994). Contraction of the CT muscle stretches the whole vocal fold stiffening both the cover and the body. On the other hand, contraction of the TA muscle tends to shorten the folds slackening the cover while imposing an internal stress on the body. Hence, the action of the CT and TA might be regarded as differential on the cover, and additive on the body. We will simply express the body stiffness as k_1=CT+TA, and the cover stiffness as k_2=CT−TA (CT and TA would represent some weighted measure of the muscle activities). At phonation in the chest register, the activity of the TA is relatively high, whereas the activity of the CT is relatively low. This combination would result in a high value for k_1 and a low value for k_2, lower than the critical value k_{2crit}, and the vocal fold oscillation would occur around the rest position. At the falsetto register, the TA muscle is deactivated, and the contraction of the CT is large. Both effects would increase k_2 to a value higher than k_{2crit}, and the oscillation would occur around the second equilibrium position, in a wider and convergent glottis.

Numerical solutions of Eq. (1) for each subregion are shown in Figs. 4 and 5, where $a_i=2l_g(x_i+x_{i0})$, i=1,2, is the glottal area.

Figure 4(a) shows the result for k_1=80 000 dyn/cm, k_2=500 dyn/cm, k_c=5000 dyn/cm, m_1=0.125 g, m_2=0.025 g, ζ_1=0.1, $\zeta_2=0.6(r_i=2\zeta_i\sqrt{m_ik_i})$, l_g=1.4 cm, d_1=0.25 cm, x_{10}=0.02 cm, x_{20}=0.0199 cm, and P_s=8000 dyn/cm^2. Except for k_2 and k_c, this is the standard configuration of the two-mass model (Ishizaka and Flanagan, 1972). k_c was given a low value for a more abrupt effect in the equilibrium position switch, and k_2 was selected lower than the transcritical value k_{2crit}=2051 dyn/cm, calculated from Eq. (8). The equilibrium position calculated from Eqs. (4) and (5) is x_1=0.00049 cm, x_2=0.00044 cm, with a slightly convergent glottis. If we calculate the medial position of the vocal folds from the plot, we obtain x_1=0.015 cm, x_2=0.019 cm, with a divergent glottis. The difference between these results is a consequence of the nonlinearity of the equations. The oscillation frequency is f=99 Hz. The plot also shows collision between the vocal folds, and a large phase difference of φ=147°. All these features correspond to a chest-like oscillation.

In (b) the cover stiffness was increased to k_2=20 000 dyn/cm, higher than the transcritical value and hence within the region associated with the falsetto, representing a simultaneous deactivation of the TA muscle and large contraction of the CT. This value of k_2 was taken near the higher limit for the oscillation region (i.e., the rest position becomes stable at higher values of k_2). The body stiffness k_1 was kept at the same value as in (a). In this case, the equilibrium

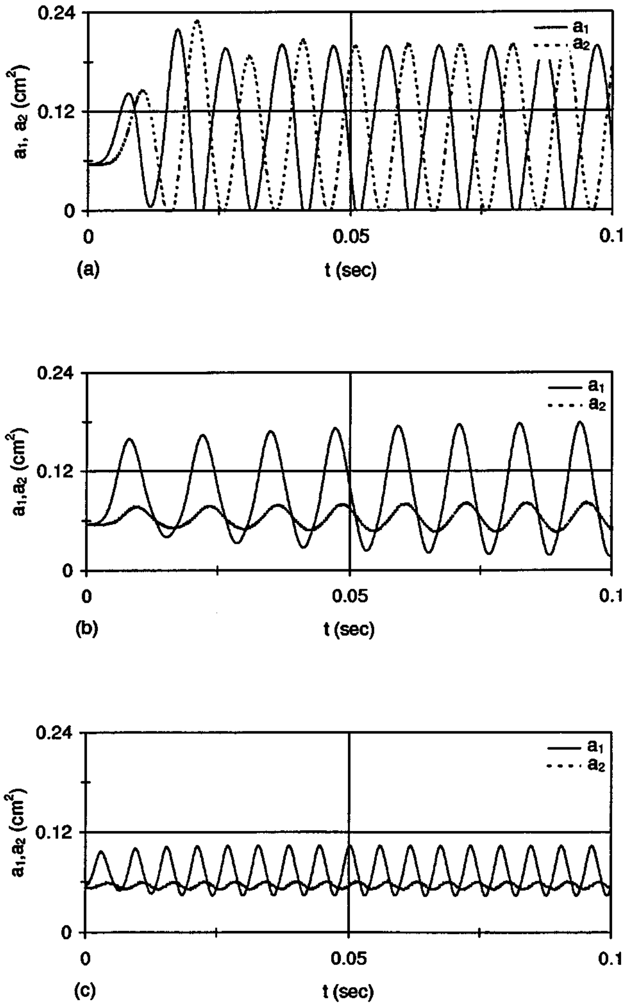

FIG. 4. Numerical solution of Eq. (1) for k_c=5000 dyn/cm, and (a) k_1=80 000 dyn/cm, k_2=500 dyn/cm$<k_{2\text{crit}}$, m_1=0.125 g, (b) k_1=80 000 dyn/cm, k_2=20 000 dyn/cm$>k_{2\text{crit}}$, m_1=0.125 g, and (c) k_1=160 000 dyn/cm, k_2=28 000 dyn/cm$>k_{2\text{crit}}$, m_1=0.05 g.

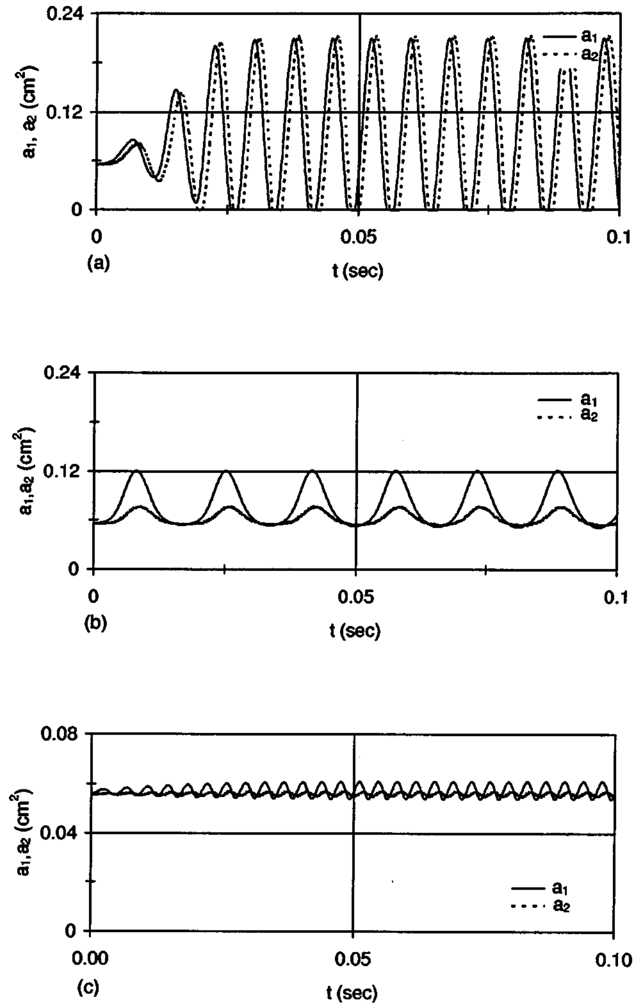

FIG. 5. Numerical solution of Eq. (1) for k_c=25 000 dyn/cm, and (a) k_1=80 000 dyn/cm, k_2=8000 dyn/cm$<k_{2\text{crit}}$, m_1=0.125 g, (b) k_1=80 000 dyn/cm, k_2=55 000 dyn/cm$>k_{2\text{crit}}$, m_1=0.125 g, and (c) k_1=160 000 dyn/cm, k_2=80 000 dyn/cm$>k_{2\text{crit}}$, m_1=0.025 g.

position is x_1=0.022 cm, x_2=0.0045 cm, which corresponds to a glottis wider and more convergent than in the previous plot. The medial position is x_1=0.015 cm, x_2=0.0030 cm. The frequency is f=86 Hz, there is no fold collision, and the phase difference decreased to φ=45°. Except for the frequency decrease, this example may be considered as a falsetto-like oscillation.

The transition to the falsetto register is normally associated with a frequency jump, instead of the decrease shown in (b). However, note that all the parameters except k_2 were kept constant. In (c) a more realistic configuration for the falsetto register is shown. As in Story and Titze (1995), we assume that the large CT contraction increases also the body stiffness k_1, to a value k_1=160 000 dyn/cm^2. Also, m_1 was reduced to m_1=0.05 g, assuming that the effective oscillating mass of the body reduces to the ligament only. The cover stiffness k_2 was also taken near the higher limit for the oscillation region, at k_2=28 000 dyn/cm^2. The transcritical value for k_2 in this case is $k_{2\text{crit}}$=6956 dyn/cm, hence this configuration is within the falsetto region $k_2>k_{2\text{crit}}$. We can see that the oscillation has falsetto-like features, with a frequency jump to f=171 Hz.

The above plots were selected as rather extreme examples of the oscillation around each equilibrium position. Note that the large phase difference in Fig. 4(a) results in the glottis opening twice in each oscillation cycle, which is not typical of the chest register, as pointed by a reviewer of this letter. In Fig. 5 another set of plots is shown, using the standard value for k_c=25 000 dyn/cm. In (a), the cover stiffness was given the standard value k_2=8000 dyn/cm, which is lower than the transcritical value $k_{2\text{crit}}$=11428 dyn/cm. The oscillation is chest-like, with a frequency f=134 Hz and phase difference φ=44°. The equilibrium position is located at x_1=0.0012 cm, x_2=0.00088 cm. In (b), k_2 was increased to a value k_2=55 000 dyn/cm, higher than $k_{2\text{crit}}$. The oscillation is falsetto-like, with a frequency f=64 Hz, and phase difference φ=13°. The equilibrium position is located at x_1=0.0142 cm, x_2=0.0044 cm, with a wider and more convergent glottis than in (a). Finally, a falsetto-like oscillation with a higher frequency of f=274 Hz is shown in (c). There, an increase of the body stiffness to k_1=160 000 dyn/cm and a decrease of its mass to m_1=0.025 g was assumed. The cover stiffness was given the value k_2=80 000 dyn/cm, which is higher than the transcritical value $k_{2crit}=42105$ dyn/cm.

A point of discussion is that, although the chest–falsetto transition consistently occurs in the frequency range 300–350 Hz (Titze, 1994), the present analysis do not show any particular relation to those frequencies.

It is to be noted that the above examples are not intended to simulate closely the chest and falsetto registers, but rather to show that each of the oscillation subregions have chest- and falsetto-like characteristics.

IV. CONCLUSION

It has been shown that chest- and falsetto-like oscillation are obtained when the two-mass model of the vocal folds is set to oscillate around different equilibrium positions. This suggests the possibility of associating each vocal register with a different equilibrium position of the vocal folds. The switch between registers would thus be caused by a transcritical bifurcation phenomenon.

The results are not conclusive, and have been presented with the intention of stimulating discussions and further research on the subject. The limitations of the two-mass model do not permit a more comprehensive treatment of vocal registers, mainly because of the difficulty in relating the model parameters to the actual vocal fold physiology. Thus analyses using more elaborated models such as the three-mass body-cover model of Story and Titze (1995) would be desirable as a next step.

One of the reviewers of this letter noted that in a previous work, Herzel (1993) found a parameter region of coexistence of limit cycles in a two-mass model, which could be related to different vocal registers. However, that region was small and at the borderline of the region for normal phonation. Further, the model used did not include the formation of a jet stream above the glottal constriction, or pressure losses for viscosity. Our results show that the coexistence of different limit cycles and other nonlinear phenomena reported in that paper do not occur when the jet formation or the viscous losses are incorporated into the model. Therefore, it is not clear whether those phenomena represent a real feature of the vocal fold oscillation, or they are just a product of the particular model used.

We believe that the study of vocal registers from a nonlinear dynamics approach along the above lines (transcritical bifurcation or coexistent limit cycles) or other alternatives is interesting and deserves further research.

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