On the relation between the phonation threshold lung pressure and the oscillation frequency of the vocal folds (L)

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This Letter presents an extension of a previous equation for the phonation threshold pressure by Titze [I. R. Titze, J. Acoust. Soc. Am. **83**, 1536–1552 (1988)]. The extended equation contains the vocal-fold oscillation frequency as an explicit factor. It is derived from the mucosal wave model of the vocal folds by considering the general case of an arbitrary time delay for the mucosal wave to travel the glottal height. The results are illustrated with a numerical example, which shows good qualitative agreement with experimental measures. © *2007 Acoustical Society of America.*
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I. INTRODUCTION

Almost two decades ago, Titze (1988) set forth the dynamical principles of the vocal-fold oscillation. He proposed a mucosal wave model in which motion of the vocal-fold tissues is represented as a surface wave propagating in the direction of the airflow. His model demonstrated the aeroelastic nature of the oscillation, born from the interaction between the airflow blowing through the glottis and the elastic structure of the tissues. Since then, the original model and its several variations have been used in further studies of phonation dynamics (e.g., Chan and Titze, 2006; Drioli, 2005; Laje *et al.*, 2001; Lucero, 1999), and have even been applied to the production of sound in the avian syrinx (Laje and Mindlin, 2005).

An important result of Titze's work was an equation for the phonation threshold value of lung pressure, defined as the minimum value required to initiate the vocal-fold oscillation. At this threshold value, the energy transferred from the airflow to the vocal folds is large enough to overcome the energy dissipated in the tissues, so that an oscillatory movement of growing amplitude may take place. Titze's equation related the threshold pressure to biomechanical parameters, namely, glottal geometry, tissue damping coefficient, and mucosal wave velocity. It has found important applications in studies of glottal aerodynamics (Titze, 1992), and its validity has been tested in various experimental setups (e.g., Chan *et al.*, 1997; Chan and Titze, 2006; Titze *et al.*, 1995). The threshold pressure value has also been interpreted as a measure of ease of phonation, and proposed as a diagnostic tool for vocal health (Titze *et al.*, 1995). Its clinical significance has been explored in several studies (e.g., Fisher and Swank, 1997; Fisher *et al.*, 2001; Milbrath and Solomon, 2003; Roy *et al.*, 2003; Sivasankar and Fisher, 2002; Verdolini *et al.*, 2002).

Naturally, the mucosal wave model included several simplifying assumptions, necessary to permit the analytical treatment. One of those assumptions was a small time delay for the mucosal wave to travel along the vertical dimension of the vocal folds. This is equivalent to a small phase delay of the oscillatory motion of the upper edge of the vocal folds in relation to the lower edge.[1] As will be shown later, a consequence of that assumption is that the equation for the phonation threshold pressure lacks the oscillation frequency as a parameter. It is well-known that phonation threshold pressure increases with frequency, as demonstrated by experimental measures (e.g., Titze, 1992). This result is consistent with data suggesting that phonation onset is delayed when speakers use higher frequencies (e.g., Koenig *et al.*, 2005). The data on how frequency affects phonation are somewhat conflicting, however (cf. McCrea and Morris, 2005). Titze (1988, 1992) pointed out the lack of a frequency factor and offered a possible solution by relating other parameters in the phonation threshold pressure expression, such as vocal-fold thickness and mucosal wave velocity, to the oscillation frequency.

This Letter will show that a more general analysis of the model allowing for arbitrary time delay results in an extended equation for the phonation threshold pressure which includes the oscillation frequency explicitly.

II. MUCOSAL WAVE MODEL

For details on the derivation of the model's equations, we refer the reader to Titze's (1988) original work. Figure 1 shows a schematic of the mucosal wave model. Complete right-left symmetry of the folds is assumed, and motion of tissues is allowed only in the horizontal direction. A surface wave propagates through the superficial tissues, in the direction of the airflow (upward).

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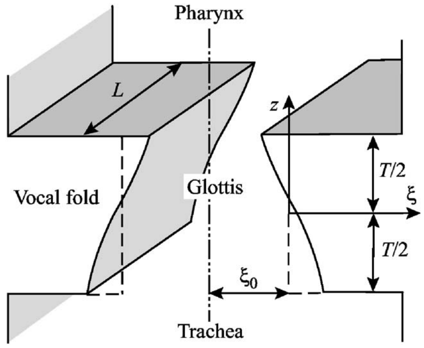

FIG. 1. Vocal-fold model (after Titze, 1988).

The equation of motion of the vocal-fold tissues is obtained by lumping their biomechanical properties at the midpoint of the glottis, and assuming that they are forced by the mean glottal pressure P_g, which yields

$$M\ddot{\xi} + B\dot{\xi} + K\xi = P_g, \tag{1}$$

where ξ is the tissue displacement at the midpoint, and M, B, K are the mass, damping, and stiffness, respectively, per unit area of the medial surface of the vocal folds.

The glottal aerodynamics is modeled by assuming that the flow is frictionless, stationary, and incompressible. Under such conditions, the mean glottal air pressure P_g may be expressed by

$$P_g = P_i + (P_s - P_i)(1 - a_2/a_1 - k_e)/k_t, \tag{2}$$

where P_s is the subglottal pressure, P_i is the supraglottal pressure (at the entry of the vocal tract), k_e is a pressure recovery coefficient for the turbulent region at the glottal exit ($0 \leqslant k_e \leqslant 0.2$, depending on the relation of the vocal-tract input area to the glottal area), k_t is a transglottal pressure coefficient ($k_t = k_c - k_e$, where k_c is a pressure loss coefficient for the region upstream the glottal exit, with values $1.0 \leqslant k_c \leqslant 1.4$ depending on the glottal channel shape), and a_1, a_2 are the glottal areas at the lower and upper edges of the glottal channel, respectively. The time-varying glottal areas are given by

$$a_1(t) = 2L(\xi_{01} + \xi(t + \tau)), \tag{3}$$

$$a_2(t) = 2L(\xi_{02} + \xi(t - \tau)), \tag{4}$$

where ξ_{01} and ξ_{02} are the lower and upper prephonatory glottal half-widths, respectively, τ is the time delay for the mucosal wave to travel half the glottal height ($T/2$ in Fig. 1), and L is the vocal-fold length. The delay τ depends on the velocity of the mucosal wave, which is related to the compliance of superficial tissues (Titze, 1992).

Following Titze, we assume that the subglottal pressure P_s is equal to a constant lung pressure P_L, the vocal-tract pressure is atmospheric ($P_i = 0$), and the vocal-tract input area is much larger than the glottal area so that $k_e \approx 0$. Further, and for simplicity of the present analysis, we consider only the case in which the prephonatory glottal channel is rectangular, i.e., the glottal half-width has a constant value $\xi_0 = \xi_{01} = \xi_{02}$ along the glottal height. Under such conditions, the mean glottal pressure simplifies to

$$P_g = \frac{P_L}{k_t}\left(1 - \frac{a_2}{a_1}\right). \tag{5}$$

Substituting into Eq. (1), we obtain the final equation for the vocal-fold oscillation

$$M\ddot{\xi} + B\dot{\xi} + K\xi = \left(\frac{P_L}{k_t}\right)\frac{\xi(t+\tau) - \xi(t-\tau)}{\xi_0 + \xi(t+\tau)}. \tag{6}$$

III. STABILITY ANALYSIS

A. Small τ approximation

Equation (6) is a functional differential equation with advance and delay arguments ($t+\tau$ and $t-\tau$, respectively). First, let us assume that the time delay τ is small enough so that the advanced-delay terms may be approximated by the linearization

$$\xi(t \pm \tau) \approx \xi(t) \pm \tau\dot{\xi}(t), \tag{7}$$

which reduces Eq. (6) to an ordinary differential equation

$$M\ddot{\xi} + B\dot{\xi} + K\xi = \left(\frac{P_L}{k_t}\right)\frac{2\tau\dot{\xi}}{\xi_0 + \xi + \tau\dot{\xi}}. \tag{8}$$

The above equation may now be analyzed by standard qualitative methods for dynamical systems. It has a unique fixed point at $\xi = 0$ (the prephonatory position). This position is stable for low values of P_L, and becomes unstable when P_L reaches the threshold value

$$P_{\text{th}} = \frac{k_t\xi_0 B}{2\tau}. \tag{9}$$

At this threshold value, the vocal-fold oscillation is generated. The oscillation threshold constitutes a Hopf bifurcation of the subcritical type, where the prephonatory position becomes unstable and at the same time absorbs an unstable limit cycle (Lucero, 1999).

B. General case for arbitrary τ

Let us consider now the general case, given by Eq. (6). The stability of the prephonatory position at $\xi = 0$ is determined by the roots of the characteristic equation associated with the linearization (variational equation) around that position (see, e.g., Hale, 1977). The linearization may be obtained by replacing the right side of Eq. (6) by the linear terms of a Taylor expansion around $\xi = 0$, which produces

$$M\ddot{\xi} + B\dot{\xi} + K\xi = \left(\frac{P_L}{k_t\xi_0}\right)[\xi(t+\tau) - \xi(t-\tau)]. \tag{10}$$

The characteristic equation may be obtained by the standard technique of proposing a solution $\xi(t) = Ce^{\lambda t}$, where C and λ are constants, and seeking nonzero solutions, which yields

$$M\lambda^2 + B\lambda + K - \frac{2P_L}{k_t\xi_0}\sinh(\lambda\tau) = 0. \tag{11}$$

For $P_L = 0$, Eq. (11) has the roots

$$\lambda = -\frac{B}{2M} \pm \sqrt{\left(\frac{B}{2M}\right)^2 - \frac{K}{M}}, \tag{12}$$

which have negative real parts. For $P_L > 0$, Eq. (11) may have an indefinite number of roots. Let us assume a pair of imaginary roots $\lambda = i\omega$. Substituting into Eq. (11), using the identity $\sinh(ix) = i\sin(x)$, and separating real and imaginary parts, we obtain

$$-\omega^2 M + K = 0, \tag{13}$$

$$\omega B - \frac{2P_L}{k_t \xi_0}\sin(\omega\tau) = 0. \tag{14}$$

The first equation produces the oscillation angular frequency $\omega = \sqrt{K/M}$. The value of P_L given by the second equation is the oscillation threshold pressure

$$P_{\text{th}} = \frac{k_t \xi_0 B \omega}{2\sin(\omega\tau)}, \tag{15}$$

with $0 < (\omega\tau) < \pi$.

According to Rouché's theorem (Dieudonné, 1960), the roots of the characteristic equation depend continuously on the parameter P_L. Hence, for $0 \leqslant P_L < P_{\text{th}}$, all roots have negative real parts, and at $P_L = P_{\text{th}}$, a pair of roots becomes imaginary. We verify next that those roots cross the imaginary axis from left to right. Implicit differentiation of Eq. (11) produces

$$\left[2M\lambda + B - \frac{2\tau P_L}{k_t \xi_0}\cosh(\lambda\tau)\right]\frac{d\lambda}{dP_L} = \frac{2}{k_t \xi_0}\sinh(\lambda\tau). \tag{16}$$

Substituting $\lambda = i\omega$, $P_L = P_{\text{th}}$, given by Eq. (15), and separating the real part, we obtain finally

$$\left.\frac{d\,\mathrm{Re}(\lambda)}{dP_L}\right|_{P_L = P_{\text{th}}} = \frac{4\omega M \sin(\omega\tau)}{k_t \xi_0 \{B^2[1 - \omega\tau\cot(\omega\tau)]^2 + 4\omega^2 M^2\}} > 0, \tag{17}$$

for $0 < (\omega\tau) < \pi$. This is the transversatility condition, which proves that the roots cross the imaginary axis and therefore their real parts become positive.

The above results imply that the equilibrium position at $\xi = 0$ is stable for $P_L < P_{\text{th}}$, and unstable for $P_L > P_{\text{th}}$. Further, by the Hopf bifurcation theorem for functional differential equations (Hale, 1977), a limit cycle is generated at $P_L = P_{\text{th}}$.

IV. PHONATION THRESHOLD PRESSURE

The phonation threshold pressure is then given by Eq. (15). Note that it now contains the oscillation frequency ω as an explicit factor. Rewriting it in the form

$$P_{\text{th}} = \frac{k_t \xi_0 B}{2\tau}\frac{\omega\tau}{\sin(\omega\tau)}, \quad 0 < (\omega\tau) < \pi, \tag{18}$$

and considering that $\sin(x)/x$ is a monotonically decreasing function in $(0, \pi)$, then we have that P_{th} increases with the oscillation frequency ω, if all other factors are fixed. Note also that, for $\tau \to 0$, $\sin(\omega\tau) \to \omega\tau$, it simplifies to Titze's result [Eq. (9)].

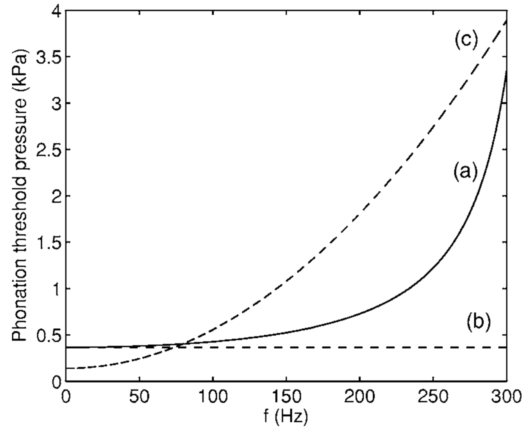

FIG. 2. Phonation threshold pressure vs oscillation frequency. (a) Value given by our extended expression Eq. (15); (b) Titze's (1988) theoretical expression, (c) Titze's (1992) empirical model.

Let us consider a numerical example, with $k_t = 1.1$, $\xi_0 = 1$ mm, $B = 1000$ Pa s/m, $c = 1$ m/s, $K = 2 \times 10^6$ Pa/m, $T = 3$ mm, $\tau = T/(2c) = 1.5$ ms (Titze, 1988). Figure 2 shows the phonation threshold pressure computed from Eq. (15), as a function of the frequency $f = \omega/(2\pi)$. For comparison, the figure also shows values from Titze's (1988) Eq. (9). That equation produces a horizontal line, since it is independent of the frequency f, and coincides with Eq. (15) at $f = 0$. Finally, it also shows results from an empirical model by Titze (1992), expressed by $P_{th} = 0.14 + 0.60(f/120)^2$. This equation was obtained by fitting a quadratic polynomial to experimental measures of phonation pressures. As the example illustrates, Eq. (15) provides a good qualitative prediction of the relation of phonation threshold pressure with oscillation frequency.

V. CONCLUSION

We have presented an equation for the phonation threshold pressure as a function of vocal-fold biomechanical parameters, which extends a previous result by Titze (1988). It contains the vocal-fold oscillation frequency as an explicit factor, and provides a good prediction for the observed increase of the threshold pressure with oscillation frequency.

The analysis was based on Titze's mucosal wave model, and considering the general case of an arbitrary time delay for the the mucosal wave to travel the glottal height. Several interesting questions appear now for further research, such as: (1) In the case of small time delay, the Hopf bifurcation at threshold is of the subcritical type (Lucero, 1999). Is it still subcritical at large time delays? (2) Titze (1992) built an empirical model of the relation between phonation threshold pressure vs frequency by fitting a quadratic polynomial to empirical data. Could a better model be obtained by fitting a function with the factor $x/\sin(x)$? (3) Our analysis assumed a rectangular prephonatory glottis, but it may be extended to a convergent-divergent shaped glottis, as in Titze's (1988) work. (4) The mucosal wave model has been used to study many aspects of phonation, including the following: the balance between the energy transferred from the airflow to the tissues and the energy dissipated (Lucero, 1999); the optimal glottal geometry for ease of phonation (Lucero, 1998); the influence of vocal-tract acoustics on phonation threshold pressure (Chan and Titze, 2006); and characteristics of labial oscillation in the avian syrinx (Laje and Mindlin, 2005). Those and other similar studies could be improved by extending them to the general case of arbitrary time delay.

Finally, let us note that the two-mass model of the vocal folds predicts a linear increase of phonation threshold pressure when the natural frequencies of the model are increased (e.g., Lucero and Koenig, 2005; Mergell *et al.*, 1999), instead of the above nonlinear relation. The consequences of that difference, and a possible way to reconcile both models is also a good subject for further analysis.

ACKNOWLEDGMENTS

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[1]This assumption was incorporated through a small angle phase approximation; see Titze (1988), p. 1540, 2nd column.

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